

Cosmological Implications of Spinor-Torsion Coupling

Nikodem J. Popławski

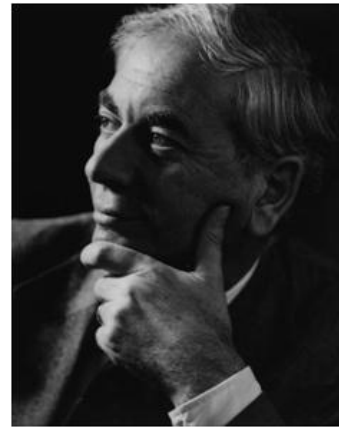
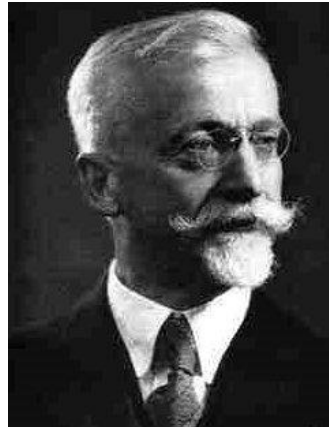
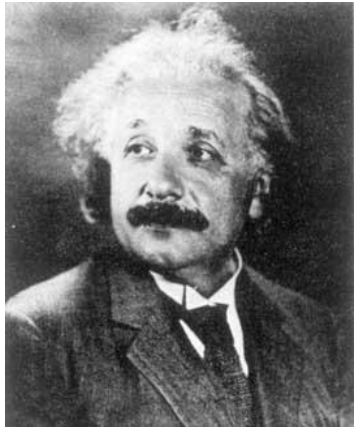
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Astrophysics-Relativity Seminar
Department of Physics and Astronomy
University of Missouri-Columbia, Columbia, MO
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Outline

1. Einstein-Cartan-Sciama-Kibble gravity
2. Spin-torsion coupling of Dirac fields
3. Spin fluids
4. Big bounce instead of big bang
5. Torsion as alternative to inflation
6. Dark energy from torsion
7. Matter-antimatter asymmetry from torsion

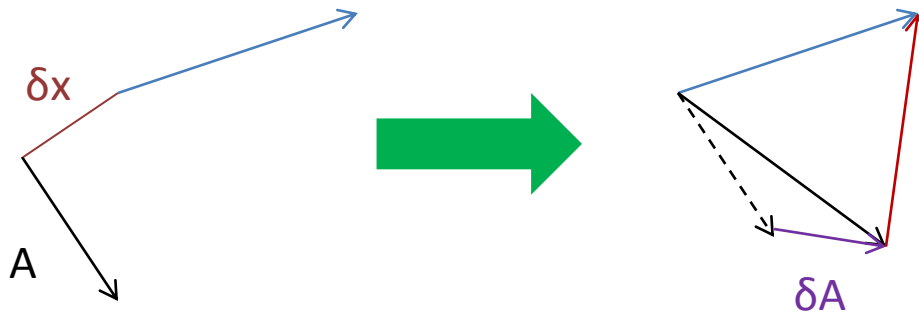
Gravity with torsion



Einstein-Cartan-Sciama-Kibble
theory of gravity

What is torsion?

- Tensors – behave under coordinate transformations like products of differentials and gradients. Special case: vectors.
- Differentiation of vectors in curved spacetime requires subtracting two infinitesimal vectors at two points that have different transformation properties.
- **Parallel transport** allows to bring one vector to the origin of the other one, so that their **difference** would make sense.



$$\delta A^i = -\Gamma^i_{jk} A^j \delta x^k$$



Affine connection

What is torsion?

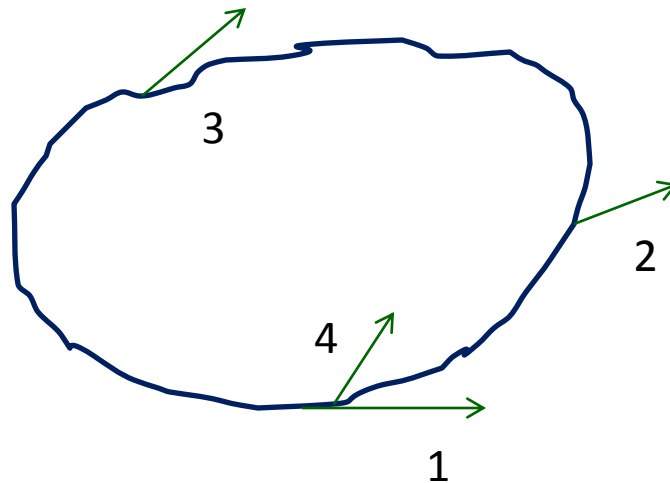
- Curved spacetime requires geometrical structure: affine connection $\Gamma^{\rho}_{\mu\nu}$

- Covariant derivative

$$\nabla_{\nu} V^{\mu} = \partial_{\nu} V^{\mu} + \Gamma^{\mu}_{\rho\nu} V^{\rho}$$

- Curvature tensor

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu} \Gamma^{\rho}_{\sigma\nu} - \partial_{\nu} \Gamma^{\rho}_{\sigma\mu} + \Gamma^{\rho}_{\tau\mu} \Gamma^{\tau}_{\sigma\nu} - \Gamma^{\rho}_{\tau\nu} \Gamma^{\tau}_{\sigma\mu}$$



Measures the change of a vector parallel-transported along a closed curve:

change = curvature X area X vector

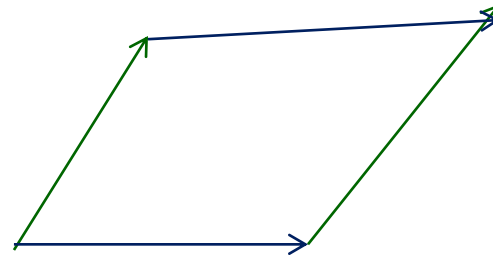
What is torsion?

- **Torsion tensor** – antisymmetric part of affine connection

$$S^k_{ij} = \Gamma_{[ij]}^k$$

- Contortion tensor

$$C^i_{jk} = S^i_{jk} + S_{jk}^i + S_{kj}^i$$



Measures noncommutativity of parallel transports

GR – affine connection restricted to be **symmetric** in lower indices

ECSK – no constraint on connection: more natural

Theories of spacetime

Special Relativity – flat spacetime (no curvature)

Dynamical variables: matter fields

$$g_{ik} = \eta_{ik}$$



General Relativity – (curvature, no torsion)

Dynamical variables: matter fields + metric tensor g_{ik}

$$S^k_{ij} = 0$$



ECSK Gravity (simplest theory with curvature & **torsion**)

Dynamical variables: matter fields + metric g_{ik} + torsion S^k_{ij}

More degrees
of freedom



ECSK gravity

T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961)
D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964)

- Riemann-Cartan spacetime – metricity $\nabla_{\rho} g_{\mu\nu} = 0$

→ connection $\Gamma^{\rho}_{\mu\nu} = \{\rho_{\mu\nu}\} + C^{\rho}_{\mu\nu}$

Christoffel symbols   contortion tensor

- Lagrangian density for matter \mathcal{L}

Metrical energy-momentum tensor

$$T_{ik} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta g^{ik}}$$

Spin tensor

$$s^{ijk} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta C_{ijk}}$$

Total Lagrangian density $-\frac{R\sqrt{-g}}{2\kappa} + \mathcal{L}$ (like in GR)

ECSK gravity

T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961)
D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964)

- Curvature tensor = Riemann tensor
+ tensor **quadratic** in torsion + total derivative
- Stationarity of action under $\delta g^{\mu\nu} \rightarrow$ Einstein equations

$$R^{\{\}_{\mu\nu}} - R^{\{\}g_{\mu\nu}}/2 = k(T_{\mu\nu} + U_{\mu\nu})$$

$$U_{\mu\nu} = [C^{\rho}_{\mu\rho} C^{\sigma}_{\nu\sigma} - C^{\rho}_{\mu\sigma} C^{\sigma}_{\nu\rho} - (C^{\rho\sigma}_{\rho} C^{\tau}_{\sigma\tau} - C^{\sigma\rho\tau} C_{\tau\rho\sigma}) g_{\mu\nu} / 2] / k$$

- Stationarity of action under $\delta C^{\mu\nu}_{\rho} \rightarrow$ Cartan equations

$$S^{\rho}_{\mu\nu} - S_{\mu} \delta^{\rho}_{\nu} + S_{\nu} \delta^{\rho}_{\mu} = -k s_{\mu\nu}^{\rho} / 2$$

Same coupling constant k

$$S_{\mu} = S^{\nu}_{\mu\nu}$$

- Cartan equations are algebraic and **linear: torsion \propto spin density**
- Contributions to energy-momentum from spin are **quadratic**

ECSK gravity

T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961)
D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964)

- Field equations with full Ricci tensor can be written as

$$R_{\mu\nu} - Rg_{\mu\nu}/2 = \Theta_{\nu\mu}$$

 Tetrad energy-momentum tensor

- Belinfante-Rosenfeld relation

$$\Theta_{\mu\nu} = T_{\mu\nu} + \nabla^*_{\rho} (s_{\mu\nu}{}^{\rho} + s^{\rho}{}_{\nu\mu} + s^{\rho}{}_{\mu\nu}) / 2$$

$$\nabla^*_{\rho} = \nabla_{\rho} - 2S_{\rho}$$

-  Conservation law for spin

$$\nabla^*_{\rho} s_{\mu\nu}{}^{\rho} = (\Theta_{\mu\nu} - \Theta_{\nu\mu})$$

-  Cyclic identities

$$R^{\sigma}{}_{\mu\nu\rho} = -2\nabla_{\mu} S^{\sigma}{}_{\nu\rho} + 4S^{\sigma}{}_{\tau\mu} S^{\tau}{}_{\nu\rho} \quad (\mu, \nu, \rho \text{ cyclically permuted})$$

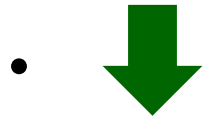
ECSK gravity

T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961)
D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964)

- Bianchi identities

(μ, ν, ρ cyclically permuted)

$$\nabla_{\mu} R^{\sigma}{}_{\tau\nu\rho} = 2R^{\sigma}{}_{\tau\pi\mu} S^{\pi}{}_{\nu\rho}$$



Conservation law for energy and momentum

$$D_{\nu} \Theta^{\mu\nu} = C_{\nu\rho}{}^{\mu} \Theta^{\nu\rho} + s_{\nu\rho\sigma} R^{\nu\rho\sigma\mu} / 2$$

$$D_{\nu} = \nabla_{\nu} \{ \}$$



Equations of motion of particles

F. W. Hehl, P. von der Heyde, G. D. Kerlick & J. M. Nester, Rev. Mod. Phys. **48**, 393 (1976)

E. A. Lord, *Tensors, Relativity and Cosmology* (McGraw-Hill, 1976)

NJP, arXiv:0911.0334

Spin-torsion coupling of spinors

- Dirac matrices $\gamma^a \gamma^b + \gamma^b \gamma^a = 2\eta^{ab} I$
- Spinor representation of Lorentz group

$$\gamma^a = \Lambda^a_b L \gamma^b L^{-1}$$

- Spinors

$$\tilde{\psi} = L \psi \quad \bar{\tilde{\psi}} = \bar{\psi} L^{-1}$$

- Covariant derivative of spinor $\psi_{;i} = \psi_{,i} - \Gamma_i \psi$

$$V^a_{|i} = V^a_{,i} + \omega^a_{bi} V^b$$

$$\gamma^a_{|i} = \omega^a_{bi} \gamma^b - [\Gamma_i, \gamma^a]$$

Metricity $\rightarrow \gamma^a_{|i} = 0 \rightarrow$

$$\Gamma_i = -\frac{1}{4} \omega_{abi} \gamma^a \gamma^b$$

$$g_{ik} = e_i^a e_k^b \eta_{ab}$$

↑
Tetrad

Spinor connection



$$\omega^i_{ak} = e^i_{a;k}$$

Spin connection

$$\omega^a_{bi} = -\omega^a_{b i}$$

$$e^i_{a|k} = e^i_{a,k} + \Gamma^i_{jk} e^j_a - \omega^b_{ak} e^i_b = 0$$

Fock-Ivanenko coefficients (1929)

Hehl-Datta equation

- Dirac Lagrangian density

$$\mathcal{L} = \frac{i\sqrt{-g}}{2} (\bar{\psi} \gamma^i \psi_{;i} - \bar{\psi}_{;i} \gamma^i \psi) - m\sqrt{-g} \bar{\psi} \psi$$

; – covariant derivative
with affine connection

: – with Christoffel symbols

- Spin density

$$s^{ijk} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta C_{ijk}}$$

Variation of C \Leftrightarrow variation of ω

Totally antisymmetric

$$s^{ijk} = \frac{i}{2} \bar{\psi} \gamma^{[i} \gamma^j \gamma^{k]} \psi = s^{[ijk]} = -e^{ijkl} s_l$$

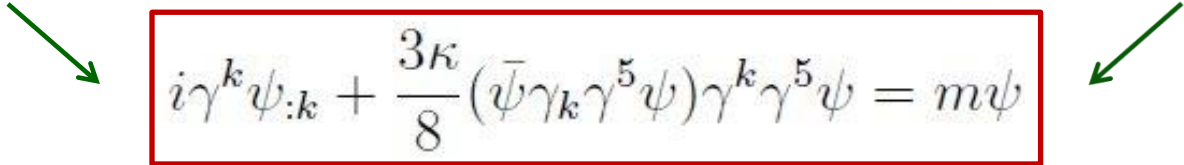
$$s^i = \frac{1}{2} \bar{\psi} \gamma^i \gamma^5 \psi \quad \text{Dirac spin pseudovector}$$

Cartan equations

$$C_{ijk} = \frac{\kappa}{4} \epsilon_{ijkl} \bar{\psi} \gamma^l \gamma^5 \psi$$

Dirac equation

$$i\gamma^k (\psi_{;k} + \frac{1}{4} C_{ijk} \gamma^i \gamma^j \psi) = m\psi$$



$$i\gamma^k \psi_{;k} + \frac{3\kappa}{8} (\bar{\psi} \gamma_k \gamma^5 \psi) \gamma^k \gamma^5 \psi = m\psi$$

Spin fluids

- Papapetrou (1951) – multipole expansion -> equations of motion

Matter in a small region in space with coordinates $x^\mu(s)$

Motion of an extended body – world tube

Motion of the body as a whole – worldline $X^\mu(s)$

- $\delta x^\alpha = x^\alpha - X^\alpha$

$$\delta x^0 = 0$$

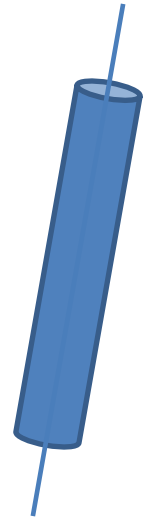
$$u^\mu = dX^\mu/ds$$

α - spatial coordinates

$$M^{\mu\nu\rho} = -u^0 \int \delta x^\mu \Theta^{\nu\rho} (-g)^{1/2} dV$$

$$N^{\mu\nu\rho} = u^0 \int s^{\mu\nu\rho} (-g)^{1/2} dV$$

Four-velocity



- Dimensions of the body small -> neglect higher-order (in δx^μ) integrals and omit surface integrals

Spin fluids

- Conservation law for spin ->

$$M^{\rho\mu\nu} - M^{\rho\nu\mu} = N^{\mu\nu\rho} - N^{\mu\nu 0} u^\rho / u^0$$

- Average fermionic matter as a continuum (fluid)

Neglect $M^{\rho\mu\nu}$ -> $s^{\mu\nu\rho} = s^{\mu\nu} u^\rho$ $s^{\mu\nu} u_\nu = 0$

Macroscopic spin tensor of a **spin fluid**

- Conservation law for energy and momentum ->

$$\Theta^{\mu\nu} = c\Pi^\mu u^\nu - p(g^{\mu\nu} - u^\mu u^\nu) \quad \epsilon = c\Pi_\mu u^\mu \quad s^2 = s^{\mu\nu} s_{\mu\nu} / 2$$

Four-momentum
density

Pressure

Energy density

J. Weyssenhoff & A. Raabe, Acta Phys. Pol. **9**, 7 (1947)

K. Nomura, T. Shirafuji & K. Hayashi, Prog. Theor. Phys. **86**, 1239 (1991)

Spin fluids

- Dynamical energy-momentum tensor for a spin fluid

$$T^{ij} + U^{ij} = \left(\epsilon - \frac{1}{4} \kappa s^2 \right) u^i u^j - \left(p - \frac{1}{4} \kappa s^2 \right) (g^{ij} - u^i u^j)$$

Energy density

Pressure

$$- (\delta_k^l + u_k u^l) D_l (s^{k(i} u^{j)})$$

for random spin orientation

F. W. Hehl, P. von der Heyde & G. D. Kerlick, Phys. Rev. D **10**, 1066 (1974)

- Barotropic fluid

$$s^2 \propto \epsilon^{2/(1+w)}$$

$$dn/n = d\epsilon/(\epsilon+p) \quad p = w\epsilon \rightarrow n \propto \epsilon^{1/(1+w)}$$

- Spin fluid of fermions with no spin polarization ->

$$s^2 = \frac{1}{8} (\hbar c n)^2$$

I. S. Nurgaliev & W. N. Ponomariev, Phys. Lett. B **130**, 378 (1983)

ECSK gravity

- No spinors \rightarrow torsion vanishes \rightarrow ECSK reduces to GR
- Torsion significant when $U_{\mu\nu} \sim T_{\mu\nu}$ (at **Cartan density**)

For fermionic matter (quarks and leptons)

$$\rho_C = \frac{m_n^2 c^4}{G \hbar^2}$$

$$\rho > 10^{45} \text{ kg m}^{-3}$$

Nuclear matter in neutron stars

$$\rho \sim 10^{17} \text{ kg m}^{-3}$$

Gravitational effects of torsion negligible even for neutron stars

Torsion significant only in very early Universe and in black holes

Cosmology with torsion

- A closed, homogeneous and isotropic Universe

Friedman-Lemaitre-Robertson-Walker metric ($k = 1$)

$$ds^2 = c^2 dt^2 - \frac{a^2(t)}{(1+kr^2/4)^2} (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2)$$

Distance from O to its antipodal point A: $a\pi$

- Friedman equations for scale factor a

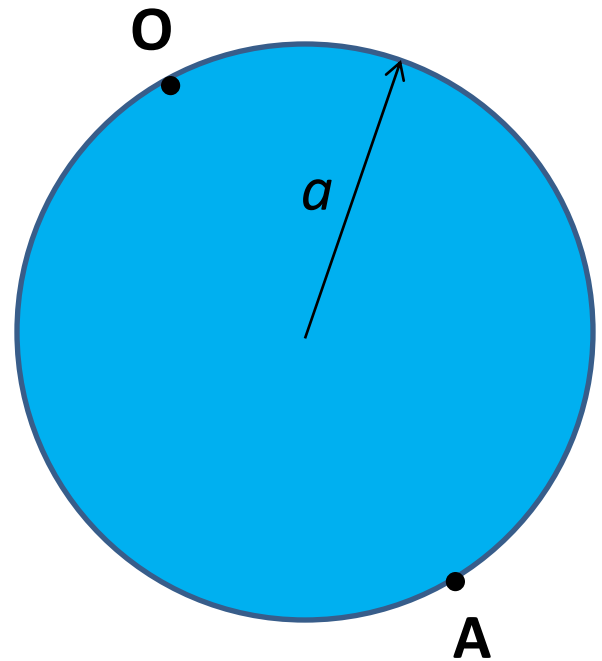
$$\dot{a}^2 + 1 = \frac{1}{3}\kappa\left(\epsilon - \frac{1}{4}\kappa s^2\right)a^2,$$

$$\dot{a}^2 + 2a\ddot{a} + 1 = -\kappa\left(p - \frac{1}{4}\kappa s^2\right)a^2$$

Conservation law



$$\frac{d}{dt}\left(\left(\epsilon - \kappa s^2/4\right)a^3\right) + \left(p - \kappa s^2/4\right)\frac{d}{dt}\left(a^3\right) = 0$$



**Spin-torsion coupling
generates
gravitational repulsion**

Cosmology with torsion

- Friedman conservation & $s^2 \propto \epsilon^{2/(1+w)} \rightarrow \epsilon \propto a^{-3(1+w)}$

Spin-torsion contribution to energy density

$$\epsilon_S = -\frac{1}{4}\kappa s^2 \propto a^{-6}$$

negative & dominant at small a

- Independent of w
- Consistent with the particle conservation $n \propto a^{-3}$

“spin fluid = perfect fluid + exotic fluid” (with $p = \epsilon = -\kappa s^2/4 < 0$)

- Very early universe: $w = 1/3$ (radiation) $\epsilon \approx \epsilon_R \sim a^{-4}$

Total energy density $\epsilon + \epsilon_S = \epsilon_{R0}\hat{a}^{-4} + \epsilon_{S0}\hat{a}^{-6}$ $\hat{a} = a/a_0$

Big bounce instead of big bang

- Friedman equation

NJP, Phys. Lett. B **694**, 181 (2010)

$$|H| = H_0 \left(\Omega_R \hat{a}^{-4} + \Omega_S \hat{a}^{-6} \right)^{\frac{1}{2}}$$

- Gravitational repulsion from spin & torsion ($\Omega_S < 0$)



No singular big bang, but regular **big bounce!** ($t = 0$)

- Universe starts expanding from **minimum radius** (when $H = 0$):

$$\hat{a}_m = \sqrt{-\frac{\Omega_S}{\Omega_R}}$$

Big bounce instead of big bang

- WMAP parameters of the Universe

$$\Omega = 1.002 \quad H_0^{-1} = 4.4 \times 10^{17} \text{ s} \quad \Omega_R = 8.8 \times 10^{-5}$$

$$\rightarrow a_0 = 2.9 \times 10^{27} \text{ m}$$

- Background neutrinos – most abundant fermions in the Universe
 $n = 5.6 \times 10^7 \text{ m}^{-3}$ for each type
- $\Omega_S = -8.6 \times 10^{-70}$ (negative, extremely small in magnitude)

$$\hat{a}_m = 3.1 \times 10^{-33}, \quad a_m = 9 \times 10^{-6} \text{ m}$$

Big bounce instead of big bang

Friedman equation in terms of density parameter

$$\Omega(\hat{a}) = 1 + \frac{(\Omega - 1)\hat{a}^4}{\Omega_R \hat{a}^2 + \Omega_S}$$

$$\dot{\hat{a}} = \frac{1}{\sqrt{\Omega(\hat{a}) - 1}}$$

Velocity of antipodal point

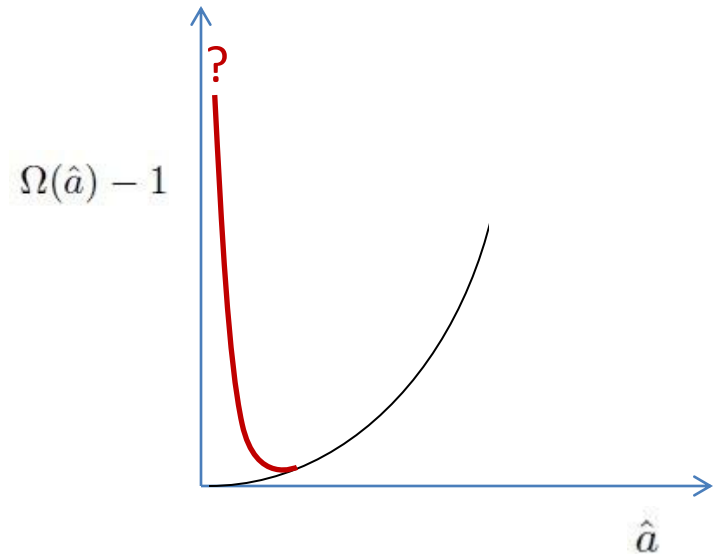
$$v_a = \pi c \dot{\hat{a}}$$

Standard cosmology

GR

$$\Omega_S = 0 \text{ and } a_m = 0$$

$$\Omega(\hat{a}) - 1 = (\Omega - 1)\hat{a}^2/\Omega_R$$



$\Omega \sim 1$ today $\rightarrow \Omega(a)$ at GUT epoch

must be tuned to 1 to a precision of > 52 decimal places

Flatness problem in big-bang cosmology, horizon problem related

Solved by **cosmic inflation** – consistent with cosmic perturbations

Problems:

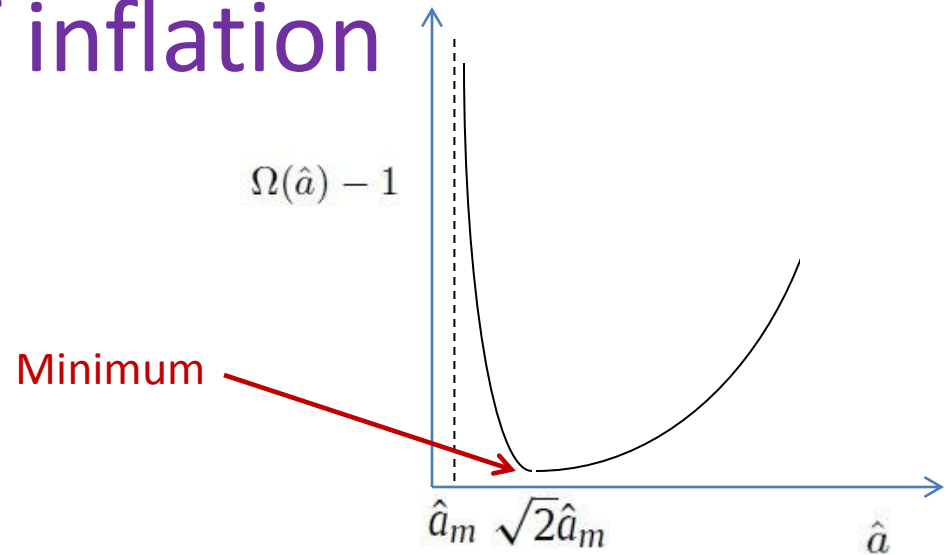
- Initial (big-bang) singularity unresolved
- Scalar field with a specific potential (slow-roll) required
- Why $\Omega \sim 1$ before inflation?
- What ends inflation?

Torsion instead of inflation

ECSK

$$\Omega_S < 0 \text{ and } a_m > 0$$

$$\Omega(\sqrt{2}\hat{a}_m) = 1 - \frac{4\Omega_S(\Omega - 1)}{\Omega_R^2}$$



$$\Omega(\sqrt{2}\hat{a}_m) = 1 + 8.9 \times 10^{-64}$$

$$t = -\frac{\Omega_S}{\Omega_R^{3/2} H_0} f(\sqrt{2}) = 5.3 \times 10^{-46} \text{ s}$$

Appears tuned to 1 to a precision of ~ 63 decimal places!

No flatness problem – advantages:

- Nonsingular bounce instead of initial singularity
- No new matter fields, additional assumptions, or free parameters
- Smooth transition: torsion epoch to radiation epoch (torsion becomes negligible)

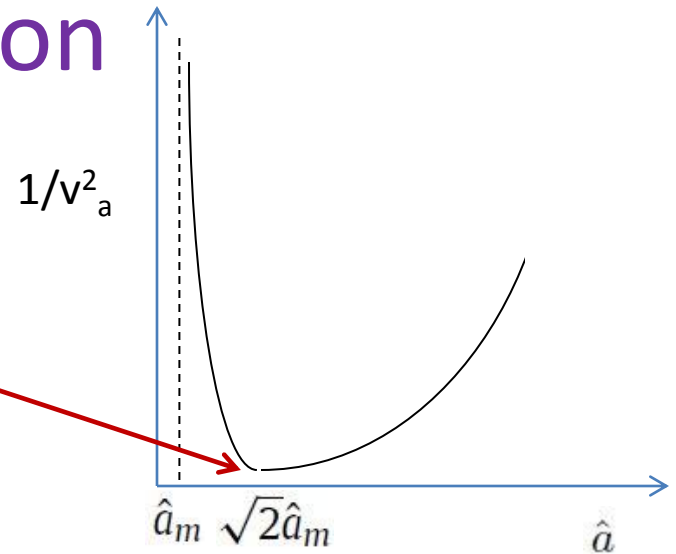
Torsion instead of inflation

ECSK

$$\Omega_S < 0 \text{ and } a_m > 0$$

$$v_a = \frac{\pi \Omega_R}{2 \sqrt{-\Omega_S(\Omega - 1)}} c = 1.1 \times 10^{32} c$$

Maximum of v_a



- Closed Universe causally connected at $t < 0$ remains causally connected through $t = 0$ until $v_a = c$
- Universe contains $N \sim (v_a/c)^3$ causally disconnected volumes
- $\Omega_S \sim -\mathbf{10}^{-69}$ produces $N \approx 10^{96}$ from a single causally connected region – torsion solves **horizon problem**

Cosmological perturbations

- Observed scale-invariant spectrum of cosmological perturbations may be produced by **thermal fluctuations** during contracting phase if background has stiff EoS

Y.-F. Cai, W. Xue, R. Brandenberger & X. Zhang, J. Cosm. Astropart. Phys. **06**, 037 (2009)

Thermal fluctuations in a contracting Universe before the big bounce = primordial fluctuations → structure formation?

Effects of Parker-Zel'dovich-Starobinskii pair production?

Work in progress

Cosmology with torsion

Physics Letters B 694 (2010) 181–185

Cosmology with torsion: An alternative to cosmic inflation

Nikodem J. Popławski

We propose a simple scenario which explains why our Universe appears spatially flat, homogeneous and isotropic. We use the Einstein–Cartan–Kibble–Sciama (ECKS) theory of gravity which naturally extends general relativity to include the spin of matter. The torsion of spacetime generates gravitational repulsion in the early Universe filled with quarks and leptons, preventing the cosmological singularity: the Universe expands from a state of minimum but finite radius. We show that the dynamics of the closed Universe immediately after this state naturally solves the flatness and horizon problems in cosmology because of an extremely small and negative torsion density parameter, $\Omega_S \approx -10^{-69}$. Thus the ECKS gravity provides a compelling alternative to speculative mechanisms of standard cosmic inflation. This scenario also suggests that the contraction of our Universe preceding the bounce at the minimum radius may correspond to the dynamics of matter inside a collapsing black hole existing in another universe, which could explain the origin of the Big Bang.

Dark energy from torsion

- Observed cosmological constant $\rho_\Lambda = (2.3 \text{ meV})^4$
- Zel'dovich formula $\rho_\Lambda \sim m^6 / m_{\text{Pl}}^2$

Y. B. Zel'dovich, J. Exp. Theor. Phys. Lett. **6**, 316 (1967)

- Spin-torsion coupling reproduces Zel'dovich formula

Effective Lagrangian density for Dirac field contains **axial-axial four-fermion interaction** (Kibble-Hehl-Datta)

$$\mathcal{L}_e = \frac{i\sqrt{-g}}{2} (\bar{\psi}\gamma^i\psi_{:i} - \bar{\psi}_{:i}\gamma^i\psi) - m\sqrt{-g}\bar{\psi}\psi + \frac{3\kappa\sqrt{-g}}{16} (\bar{\psi}\gamma_k\gamma^5\psi)(\bar{\psi}\gamma^k\gamma^5\psi)$$

$$T_{ik} = \frac{i}{2} (\bar{\psi}\delta_{(i}^j\gamma_{k)}\psi_{:j} - \bar{\psi}_{:j}\delta_{(i}^j\gamma_{k)}\psi) - \frac{i}{2} (\bar{\psi}\gamma^j\psi_{:j} - \bar{\psi}_{:j}\gamma^j\psi)g_{ik} + m\bar{\psi}\psi g_{ik} \\ - \frac{3\kappa}{16} (\bar{\psi}\gamma_j\gamma^5\psi)(\bar{\psi}\gamma^j\gamma^5\psi)g_{ik}.$$

Dark energy from torsion

HD energy-momentum tensor

$$T_{ik} = \frac{i}{2} (\bar{\psi} \delta_{(i}^j \gamma_{k)} \psi_{:j} - \bar{\psi}_{:j} \delta_{(i}^j \gamma_{k)} \psi) + \frac{3\kappa}{16} (\bar{\psi} \gamma_j \gamma^5 \psi) (\bar{\psi} \gamma^j \gamma^5 \psi) g_{ik}$$

↓
GR part

↓
cosmological term

Effective cosmological constant

$$\Lambda = \frac{3\kappa^2}{16} (\bar{\psi} \gamma_j \gamma^5 \psi) (\bar{\psi} \gamma^j \gamma^5 \psi)$$

NJP, Annalen Phys. **523**, 291 (2011)

Vacuum energy density

$$\rho_\Lambda = \frac{3\kappa}{16} (\bar{\psi} \gamma_j \gamma^5 \psi) (\bar{\psi} \gamma^j \gamma^5 \psi)$$

Not constant in time, but constant in space at cosmological distances for homogeneous and isotropic Universe

Dark energy from torsion

Cosmological constant if spinor field forms condensate with **nonzero vacuum expectation value** like in QCD

$$\langle 0 | \bar{\psi} \psi | 0 \rangle \approx -(230 \text{ MeV})^3$$

Vacuum-state-dominance approximation

M. A. Shifman, A. I. Vainshtein & V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979)

$$\langle 0 | \bar{\psi} \Gamma_1 \psi \bar{\psi} \Gamma_2 \psi | 0 \rangle = \frac{1}{12^2} \left((\text{tr} \Gamma_1 \cdot \text{tr} \Gamma_2) - \text{tr}(\Gamma_1 \Gamma_2) \right) (\langle 0 | \bar{\psi} \psi | 0 \rangle)^2$$

For quark fields

$$\langle 0 | (\bar{\psi} \gamma_j \gamma^5 t^a \psi) (\bar{\psi} \gamma^j \gamma^5 t^a \psi) | 0 \rangle = \frac{16}{9} (\langle 0 | \bar{\psi} \psi | 0 \rangle)^2$$

Axial vector-axial vector form of HD four-fermion interaction gives **positive cosmological constant**

Dark energy from torsion

Cosmological constant from QCD vacuum and ECKS torsion

$$\langle 0 | \rho_\Lambda | 0 \rangle = \frac{\kappa}{3} (\langle 0 | \bar{\psi} \psi | 0 \rangle)^2 \approx (54 \text{ meV})^4$$

$$\rho_\Lambda \sim m^6 / m_{\text{Pl}}^2$$

(Zel'dovich)

This value would agree with observations if

$$\langle 0 | \bar{\psi} \psi | 0 \rangle \approx -(28 \text{ MeV})^3$$

- Energy scale of torsion-induced cosmological constant from QCD vacuum only ~ 8 times larger than observed
- Contribution from spinor fields with lower VEV like neutrino condensates could decrease average $\langle 0 | \bar{\psi} \psi | 0 \rangle$ such that torsion-induced cosmological constant would agree with observations
- **Simplest** model predicting **positive** cosmological constant and \sim its **energy scale** – does not use new fields

Dark energy from torsion

Ann. Phys. (Berlin) 523, No. 4, 291 – 295 (2011)

Cosmological constant from quarks and torsion

Nikodem J. Popławski*

We present a simple and natural way to derive the observed small, positive cosmological constant from the gravitational interaction of condensing fermions. In the Riemann-Cartan spacetime, torsion gives rise to the axial-axial vector four-fermion interaction term in the Dirac Lagrangian for spinor fields. We show that this nonlinear term acts like a cosmological constant if these fields have a nonzero vacuum expectation value. For quark fields in QCD, such a torsion-induced cosmological constant is positive and its energy scale is only about 8 times larger than the observed value. Adding leptons to this picture could lower this scale to the observed value.

Dark energy from torsion

arXiv.org > hep-ph > arXiv:1008.0033

Emergent Photons and Gravitons: The Problem of Vacuum Structure ^{*†}

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We discuss vacuum condensates associated with emergent QED and with torsion, as well as the possible role of the Kodama wave function in quantum cosmology.

At this meeting I learned of closely related work of Poplawski.^[18] He uses the QCD quark vacuum condensates instead of a Lorentz-violating axial condensate to arrive at a very similar endpoint. This very interesting work is evidently much more conservative in nature than my utilization of a Lorentz-violating condensate; hence it is more credible.

Matter-antimatter asymmetry

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- Hehl-Datta equation

$$ie_a^\mu \gamma^a \psi_{;\mu} + qe_a^\mu A_\mu \gamma^a \psi = m \psi - \frac{3\kappa}{8} (\bar{\psi} \gamma^5 \gamma_a \psi) \gamma^5 \gamma^a \psi.$$

$$\bar{\psi} = \psi^\dagger \gamma^0$$

Adjoint spinor

- Charge conjugate

$$\psi^c = -i\gamma^2 \psi^*$$

Satisfies Hehl-Datta equation with opposite charge and **different sign for the cubic term**

$$ie_a^\mu \gamma^a \psi^c_{;\mu} - qe_a^\mu A_\mu \gamma^a \psi^c = m \psi^c + \frac{3\kappa}{8} (\bar{\psi}^c \gamma^5 \gamma_a \psi^c) \gamma^5 \gamma^a \psi^c.$$

Energy levels (effective masses)

Fermions $\omega = m + \alpha \kappa N$ (-> NR)

Antifermions $\omega = m - \alpha \kappa N$



Inverse normalization for spinor wave function

HD asymmetry significant when torsion is
-> baryogenesis -> dark matter?

Matter-antimatter asymmetry

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Matter-antimatter asymmetry and dark matter from torsion

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We propose a simple scenario which explains the observed matter-antimatter imbalance and the origin of dark matter in the Universe. We use the Einstein-Cartan-Sciama-Kibble theory of gravity which naturally extends general relativity to include the intrinsic spin of matter. Spacetime torsion produced by spin generates, in the classical Dirac equation, the Hehl-Datta term which is cubic in spinor fields. We show that under a charge-conjugation transformation this term changes sign relative to the mass term. A classical Dirac spinor and its charge conjugate therefore satisfy different field equations. Fermions in the presence of torsion have higher energy levels than antifermions, which leads to their decay asymmetry. Such a difference is significant only at extremely high densities that existed in the very early Universe. We propose that this difference caused a mechanism, according to which heavy fermions existing in such a Universe and carrying the baryon number decayed mostly to normal matter, whereas their antiparticles decayed mostly to hidden antimatter which forms dark matter. The conserved total baryon number of the Universe remained zero.

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Summary

Thank you!

- ❑ The Einstein-Cartan-Kibble-Sciama gravity accounts for spin of elementary particles, which equips spacetime with **torsion**.
- ❑ For fermionic matter at very high densities, torsion manifests itself as gravitational repulsion that prevents the formation of singularities in black holes and at big bang.
- ❑ Big-bounce cosmology with torsion solves flatness and horizon problems without inflation.
- ❑ Spinor-torsion coupling in fermions can be the origin of the cosmological constant and of the matter-antimatter asymmetry in the Universe.
- ❑ Future work
 - Cosmological perturbations in big-bounce cosmology with torsion
 - Dark matter from the spinor-torsion coupling
 - Gravitational collapse in black holes in the ECSK gravity