Cosmological Constant and Torsion

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Simplest explanation of current cosmic acceleration (dark energy) – positive Cosmological Constant (or vacuum energy density)

Measured value: $\rho_\Lambda = (2.3\,\mathrm{meV})^4$

Quantum field theory – zero-point energy of vacuum: $\rho_\Lambda \sim m_{\mathrm{Pl}}^4$
120 orders of magnitude larger than observed – very bad

Zel'dovich: $\rho_\Lambda \sim m^6/m_{\mathrm{Pl}}^2$ cosmological constant from particle physics (dimensional arguments)

Arguments for Λ = 0 – before cosmic acceleration discovered
(Hawking; Linde; Coleman)

Huge cosmological constant from zero-point energy of vacuum may be reduced via dynamical processes
(Abbott; Brown & Teitelboim; Steinhardt & Turok; Klinkhamer & Volovik)

Or: Λ can be simply another constant of Nature

Cosmological constant from particle physics (examples)

- QCD trace anomaly from gluon and quark condensates

$\rho_\Lambda \sim H\lambda_{\mathrm{QCD}}^3 \quad \lambda_{\mathrm{QCD}} \approx 200\,\mathrm{MeV}$ (Schutzhold)

- QCD gluon condensate (Klinkhamer & Volovik)

$\rho_\Lambda \sim \lambda_{\mathrm{QCD}}^6/m_{\mathrm{Pl}}^2$ resembles Zel’dovich formula

- QCD chiral quark condensate (Urban & Zhitnitsky)

$\rho_\Lambda \sim H m_q \langle q\bar{q}\rangle / m_{\eta'}$

- Electroweak phase transition (Klinkhamer & Volovik)

$\rho_\Lambda \sim E_{\mathrm{EW}}^8/m_{\mathrm{Pl}}^4$

- BCS condensate of fermions and torsion

(Alexander, Biswas & Calcagni)

Einstein-Cartan gravity

- Naturally extends GR to include matter with spin

- Spin produces torsion (antisymmetric part of affine connection); torsion proportional to spin density

- EC gravity provides more complete account of local gauge invariance with respect to Poincare group

- Viable theory of gravity; differs significantly from GR only at densities much larger than nuclear matter density

- May prevent formation of singularities from fermionic matter (which builds all stars)

NJ Poplawski, Phys. Lett. B, in press (2010)

- Requires fermions to be extended, introducing effective ultraviolet cutoff in QFT

Dirac Lagrangian and torsion → Heisenberg-Ivanenko equation (semicolon – GR covariant derivative)

$$i\gamma^k\psi_{;k} = m\psi - \frac{3\kappa}{8}(\bar{\psi}\gamma_k\gamma^5\psi)\gamma^k\gamma^5\psi$$

Effective metric Lagrangian for H-I equation

$$\mathfrak{L} = \frac{i\sqrt{-g}}{2}(\bar{\psi}\gamma^i\psi_{;i} - \bar{\psi}_{;i}\gamma^i\psi) - m\sqrt{-g}\bar{\psi}\psi + \frac{3\kappa\sqrt{-g}}{16}(\bar{\psi}\gamma_k\gamma^5\psi)(\bar{\psi}\gamma^k\gamma^5\psi)$$

Energy-momentum tensor for H-I Lagrangian

$$T_{ik} = \frac{i}{2}(\bar{\psi}\delta^j_{(i}\gamma_{k)}\psi_{;j} - \bar{\psi}_{;j}\delta^j_{(i}\gamma_{k)}\psi) - \frac{i}{2}(\bar{\psi}\gamma^j\psi_{;j} - \bar{\psi}_{;j}\gamma^j\psi)g_{ik}$$
$$+ m\bar{\psi}\psi g_{ik} - \frac{3\kappa}{16}(\bar{\psi}\gamma_j\gamma^5\psi)(\bar{\psi}\gamma^j\gamma^5\psi)g_{ik}$$

H-I energy-momentum tensor

$$T_{ik} = \underset{\substack{\big\downarrow \\ \text{GR part}}}{\frac{i}{2}(\bar{\psi}\delta^j_{(i}\gamma_{k)}\psi_{;j} - \bar{\psi}_{;j}\delta^j_{(i}\gamma_{k)}\psi)} + \underset{\substack{\big\downarrow \\ \text{cosmological term}}}{\frac{3\kappa}{16}(\bar{\psi}\gamma_j\gamma^5\psi)(\bar{\psi}\gamma^j\gamma^5\psi)g_{ik}}$$

Effective cosmological constant

$$\Lambda = \frac{3\kappa^2}{16}(\bar{\psi}\gamma_j\gamma^5\psi)(\bar{\psi}\gamma^j\gamma^5\psi)$$

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Vacuum energy density

$$\rho_\Lambda = \frac{3\kappa}{16}(\bar{\psi}\gamma_j\gamma^5\psi)(\bar{\psi}\gamma^j\gamma^5\psi)$$

Not constant in time, but constant in space at cosmological distances for homogeneous and isotropic Universe

Cosmological constant if spinor field forms condensate with nonzero vacuum expectation value, e.g., in QCD

$$\langle 0|\bar{\psi}\psi|0\rangle \approx -(230\,\mathrm{MeV})^3$$

Vacuum-state-dominance approximation
(Shifman, Vainshtein and Zakharov)

$$\langle 0|\bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi|0\rangle = \frac{1}{12^2}\Big((tr\Gamma_1 \cdot tr\Gamma_2) - tr(\Gamma_1\Gamma_2)\Big)(\langle 0|\bar{\psi}\psi|0\rangle)^2$$

For quark fields

$$\langle 0|(\bar{\psi}\gamma_j\gamma^5 t^a\psi)(\bar{\psi}\gamma^j\gamma^5 t^a\psi)|0\rangle = \frac{16}{9}(\langle 0|\bar{\psi}\psi|0\rangle)^2$$

Axial vector-axial vector form of H-I four-fermion interaction gives positive cosmological constant

Cosmological constant from QCD vacuum and EC torsion

$$\langle 0|\rho_\Lambda|0\rangle = \frac{\kappa}{3}(\langle 0|\bar{\psi}\psi|0\rangle)^2 \approx (54\,\mathrm{meV})^4$$

reproduces Zel'dovich formula

This value would agree with observations if

$$\langle 0|\bar{\psi}\psi|0\rangle \approx -(28\,\mathrm{MeV})^3$$

- Energy scale of torsion-induced cosmological constant from QCD vacuum only ~ 8 times larger than observed

- Contribution from spinor fields with lower VEV, e.g. neutrino condensates could decrease average $\langle 0|\bar{\psi}\psi|0\rangle$ such that torsion-induced cosmological constant would agree with observations

- **Simplest** model predicting positive cosmological constant and ~ its energy scale; does not use new fields