Dark Energy and Electromagnetism in Purely Affine Gravity

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WHY AFFINE GRAVITY ?

- Metric tensor as fundamental variable (Einstein–Hilbert variational principle) : metric theory of gravity (GR)
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- Λ and EM together have a simple Lagrangian only in one gravity

PURELY AFFINE GRAVITY

Affine connection	$\Gamma^{\;\rho}_{\mu\;\nu}$
Ricci tensor	$R_{\mu\nu}$
Lagrangian density	$\mathfrak{L}(\Gamma^{\;\rho}_{\mu\;\nu}, R_{\mu\nu})$
Free Lagrangian density	$\mathfrak{L}(R_{\mu\nu})$
Metric density	$\mathrm{g}^{\mu\nu} = -2\kappa\frac{\partial\mathfrak{L}}{\partial R_{\mu\nu}}$
Metric tensor	$g^{\mu\nu} = \frac{\mathrm{g}^{(\mu\nu)}}{\sqrt{-\det\mathrm{g}^{(\rho\sigma)}}}$

	PURELY AFFINE GRAVITY	MECHANICS
Affine connection	$\Gamma_{\mu\,\nu}^{\,\rho}$	q_i
Ricci tensor	$R_{\mu\nu}$	$\dot{q}_i$
Lagrangian density	$\mathfrak{L}(\Gamma_{\mu\,\nu}^{\,\rho}, R_{\mu\nu})$	$L(q_i, \dot{q}_i)$
Free Lagrangian density	$\mathfrak{L}(R_{\mu\nu})$	$L(\dot{q}_i)$
Metric density	$\mathrm{g}^{\mu\nu} = -2\kappa \frac{\partial \mathfrak{L}}{\partial R_{\mu\nu}}$	$p_i = \frac{\partial L}{\partial \dot{q}_i}$
Metric tensor	$g^{\mu\nu} = \frac{\mathrm{g}^{(\mu\nu)}}{\sqrt{-\mathrm{det}\mathrm{g}^{(\rho\sigma)}}}$	

PURELY AFFINE GRAVITY – DYNAMICS

Action variation for free Lagrangian

$$\delta S = -\frac{1}{2\kappa c}\int d^4x\, \mathrm{g}^{\mu\nu}\,\delta R_{\mu\nu}$$

Lagrange field equations

$$\mathrm{g}^{\mu\nu}{}_{,\rho} + {}^{*}\Gamma^{\ \mu}_{\sigma\ \rho}\mathrm{g}^{\sigma\nu} + {}^{*}\Gamma^{\ \nu}_{\rho\ \sigma}\mathrm{g}^{\mu\sigma} - {}^{*}\Gamma^{\ \sigma}_{\sigma\ \rho}\mathrm{g}^{\mu\nu} = 0$$

$${}^{*}\Gamma^{\ \rho}_{\mu\ \nu} = \Gamma^{\ \rho}_{\mu\ \nu} + \frac{2}{3}\delta^{\rho}_{\mu}S_{\nu}$$

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If $\mathfrak{L}$ depends only on $R_{(\mu\nu)}$ then $\mathrm{g}^{\mu\nu}$ is symmetric

$$\rightarrow\; {}^*\Gamma^{\;\rho}_{\mu\;\nu} = \left\{ {}^{\;\rho}_{\mu\;\nu} \right\}_g$$

EQUIVALENCE OF AFFINE AND METRIC GRAVITY

Affine Hamiltonian via Legendre transformation (Kijowski, GRG 1978)

$$\mathfrak{H}(\Gamma^{\,\rho}_{\mu\,\nu}, \mathrm{g}^{\mu\nu}) = \mathfrak{L} - \frac{\partial \mathfrak{L}}{\partial R_{\mu\nu}} R_{\mu\nu} = \mathfrak{L} + \frac{1}{2\kappa} \mathrm{g}^{\mu\nu} R_{\mu\nu}$$

$$H = p_i \dot{q}_i - L$$

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→ Einstein equations

$$R_{(\mu\nu)} = 2\kappa \frac{\partial \mathfrak{H}}{\partial \mathrm{g}^{\mu\nu}} \qquad \dot{q}_i = \frac{\partial H}{\partial p_i}$$

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Second Hamilton equation $${}^{*}\Gamma^{\ \rho}_{\mu\ \nu} = \left\{ {}^{\ \rho}_{\mu\ \nu} \right\}_g \qquad -\dot{p}_i = \frac{\partial H}{\partial q_i}$$

COSMOLOGICAL CONSTANT

Eddington affine Lagrangian

$$\mathfrak{L}_\Lambda = \frac{1}{\kappa\Lambda}\sqrt{-\det R_{(\mu\nu)}} \qquad\qquad L \sim \dot{q}^2$$

Algebraic relation R(g)

$$R_{(\mu\nu)} = -\Lambda g_{\mu\nu}$$

Einstein equations with cosmological constant

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$$\mathfrak{H} = \mathfrak{H}_\Lambda \quad \Longleftrightarrow \quad \mathfrak{L} = \mathfrak{L}_\Lambda$$

ELECTROMAGNETIC FIELD

Metric Maxwell Lagrangian

$$\mathfrak{H}_{EM} = -\frac{1}{4}\sqrt{-g}F_{\alpha\beta}F_{\rho\sigma}g^{\alpha\rho}g^{\beta\sigma}$$

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One can show (Ferraris and Kijowski, LMP 1981; gr-qc/0701176)

$$\mathfrak{H} = \mathfrak{H}_{EM} \Longleftrightarrow \mathfrak{L} = \mathfrak{L}_{EM}$$

Both simple Lagrangians are dynamically equivalent

ELECTROMAGNETIC FIELD + COSMOLOGICAL CONSTANT

$$\mathfrak{H} = \mathfrak{H}_{EM} + \mathfrak{H}_{\Lambda} \quad \overset{?}{\longleftrightarrow} \quad \mathfrak{L} = \mathfrak{L}_{EM} + \mathfrak{L}_{\Lambda}$$

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The sum of the two simple metric Lagrangians corresponding to cosmological constant and electromagnetic field is not dynamically equivalent to the sum of the two simple affine Lagrangians for Λ and EM.

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Two different models:

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Possible explanation of the **Pioneer anomaly** ?