

Dark Energy and Electromagnetism in Purely Affine Gravity

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WHY AFFINE GRAVITY ?

- Metric tensor as fundamental variable (Einstein–Hilbert variational principle) : **metric** theory of gravity (GR)
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- Metric Lagrangians for cosmological constant and electromagnetic field have **simple** equivalents in affine gravity
- **Λ and EM together have a simple Lagrangian only in one gravity**

PURELY AFFINE GRAVITY

Affine connection $\Gamma_{\mu\nu}^{\rho}$

Ricci tensor $R_{\mu\nu}$

Lagrangian density $\mathfrak{L}(\Gamma_{\mu\nu}^{\rho}, R_{\mu\nu})$

Free Lagrangian density $\mathfrak{L}(R_{\mu\nu})$

Metric density $g^{\mu\nu} = -2\kappa \frac{\partial \mathfrak{L}}{\partial R_{\mu\nu}}$

Metric tensor $g^{\mu\nu} = \frac{g^{(\mu\nu)}}{\sqrt{-\det g^{(\rho\sigma)}}}$

PURELY AFFINE GRAVITY

MECHANICS

Affine connection

$$\Gamma_{\mu\nu}^{\rho}$$

$$q_i$$

Ricci tensor

$$R_{\mu\nu}$$

$$\dot{q}_i$$

Lagrangian density

$$\mathfrak{L}(\Gamma_{\mu\nu}^{\rho}, R_{\mu\nu})$$

$$L(q_i, \dot{q}_i)$$

Free Lagrangian density

$$\mathfrak{L}(R_{\mu\nu})$$

$$L(\dot{q}_i)$$

Metric density

$$g^{\mu\nu} = -2\kappa \frac{\partial \mathfrak{L}}{\partial R_{\mu\nu}}$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

Metric tensor

$$g^{\mu\nu} = \frac{g^{(\mu\nu)}}{\sqrt{-\det g^{(\rho\sigma)}}}$$

PURELY AFFINE GRAVITY – DYNAMICS

Action variation for free Lagrangian

$$\delta S = -\frac{1}{2\kappa c} \int d^4x g^{\mu\nu} \delta R_{\mu\nu}$$

Lagrange field equations

$$g^{\mu\nu}{}_{,\rho} + {}^*\Gamma_{\sigma\rho}^{\mu} g^{\sigma\nu} + {}^*\Gamma_{\rho\sigma}^{\nu} g^{\mu\sigma} - {}^*\Gamma_{\sigma\rho}^{\sigma} g^{\mu\nu} = 0$$

$${}^*\Gamma_{\mu\nu}^{\rho} = \Gamma_{\mu\nu}^{\rho} + \frac{2}{3} \delta_{\mu}^{\rho} S_{\nu}$$

give differential relation $\Gamma(g)$

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$\Gamma(g) + R(\Gamma) + \text{algebraic } R(g) \rightarrow \text{differential equation for } g$

If \mathfrak{L} depends only on $R_{(\mu\nu)}$ then $g^{\mu\nu}$ is symmetric

$$\rightarrow {}^*\Gamma_{\mu\nu}^{\rho} = \left\{ \begin{matrix} \rho \\ \mu \nu \end{matrix} \right\} g$$

EQUIVALENCE OF AFFINE AND METRIC GRAVITY

Affine **Hamiltonian** via Legendre transformation (Kijowski, GRG 1978)

$$\mathfrak{H}(\Gamma_{\mu\nu}^{\rho}, g^{\mu\nu}) = \mathfrak{L} - \frac{\partial \mathfrak{L}}{\partial R_{\mu\nu}} R_{\mu\nu} = \mathfrak{L} + \frac{1}{2\kappa} g^{\mu\nu} R_{\mu\nu}$$

$$H = p_i \dot{q}_i - L$$

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First **Hamilton** equation
→ **Einstein** equations

$$R_{(\mu\nu)} = 2\kappa \frac{\partial \mathfrak{H}}{\partial g^{\mu\nu}} \quad \dot{q}_i = \frac{\partial H}{\partial p_i}$$

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$$2\kappa \delta \mathfrak{H} = R_{(\mu\nu)} \delta g^{\mu\nu} = \left(R_{(\mu\nu)} - \frac{1}{2} R g_{\mu\nu} \right) \sqrt{-g} \delta g^{\mu\nu}$$

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$$\text{Second **Hamilton** equation} \quad {}^* \Gamma_{\mu\nu}^{\rho} = \left\{ \begin{matrix} \rho \\ \mu \nu \end{matrix} \right\}_g \quad -\dot{p}_i = \frac{\partial H}{\partial q_i}$$

COSMOLOGICAL CONSTANT

Eddington affine Lagrangian

$$\mathfrak{L}_\Lambda = \frac{1}{\kappa\Lambda} \sqrt{-\det R_{(\mu\nu)}} \quad L \sim \dot{q}^2$$

Algebraic relation $R(g)$

$$R_{(\mu\nu)} = -\Lambda g_{\mu\nu}$$

Einstein equations with **cosmological** constant

Eddington Hamiltonian $\mathfrak{H}_\Lambda = -\frac{\Lambda}{\kappa} \sqrt{-g} \Leftrightarrow$ GR Lagrangian

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$$\mathfrak{H} = \mathfrak{H}_\Lambda \quad \longleftrightarrow \quad \mathfrak{L} = \mathfrak{L}_\Lambda$$

ELECTROMAGNETIC FIELD

Metric **Maxwell** Lagrangian

$$\mathfrak{H}_{EM} = -\frac{1}{4} \sqrt{-g} F_{\alpha\beta} F_{\rho\sigma} g^{\alpha\rho} g^{\beta\sigma}$$

Affine **Ferraris–Kijowski** Lagrangian

$P^{\mu\nu}$ reciprocal to $R_{(\mu\nu)}$

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One can show (Ferraris and Kijowski, LMP 1981; [gr-qc/0701176](https://arxiv.org/abs/gr-qc/0701176))

$$\mathfrak{H} = \mathfrak{H}_{EM} \longleftrightarrow \mathfrak{L} = \mathfrak{L}_{EM}$$

Both simple Lagrangians are dynamically equivalent

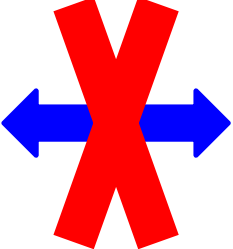
ELECTROMAGNETIC FIELD + COSMOLOGICAL CONSTANT

$$\mathfrak{H} = \mathfrak{H}_{EM} + \mathfrak{H}_{\Lambda} \quad \longleftrightarrow \quad \mathfrak{L} = \mathfrak{L}_{EM} + \mathfrak{L}_{\Lambda}$$

?

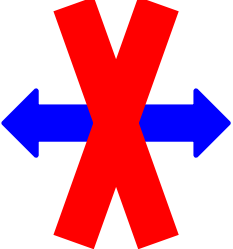
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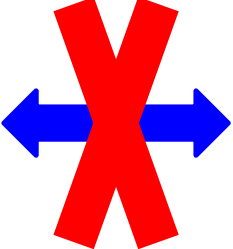
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The sum of the two **simple** metric Lagrangians corresponding to cosmological constant and electromagnetic field is **not** dynamically equivalent to the sum of the two **simple** affine Lagrangians for Λ and EM.

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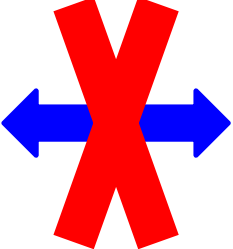
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Two different models:

1. $\mathfrak{H} = \mathfrak{H}_{EM} + \mathfrak{H}_{\Lambda}$ simple \mathfrak{L} complicated
2. $\mathfrak{L} = \mathfrak{L}_{EM} + \mathfrak{L}_{\Lambda}$ simple \mathfrak{H} complicated

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Two different models: **which one is physical ?**

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- Largest deviation between the two models occurs when $\kappa B^2 \sim \Lambda$.

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- We need the equations of motion in purely affine gravity:

$$S = -\frac{mc}{\sqrt{\Lambda}} \int \sqrt{R_{\mu\nu} dx^\mu dx^\nu} - \frac{e}{c} \int A_\mu dx^\mu \quad ?$$

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Possible explanation of the **Pioneer anomaly** ?